Frequencies of radially symmetric excitations in vortex state disks

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Submicron ferromagnetic disks with a vortex ground-state exhibit interesting and potentially useful dynamic properties arising from excitations on the ground state. In particular, a magnetic pulse applied perpendicular to the vortex plane will excite radially symmetric modes. Previous calculations of the frequencies of these modes based on the linearized Landau-Lifschitz equation and the magnetostatic Green's function give eigenfrequencies proportional to the square root of the aspect ratio $\sqrt{L/R}$, where L is the disk thickness and R is the disk radius. However, experimental frequency data show significant deviation from the square-root dependence. An improved calculation of the frequency is done through a collective variable approach by exploiting the high symmetry of these modes. In the linear approximation, this approach gives the main contribution to the frequency proportional to $\sqrt{(L/R)\ln(R/L)}$, which is closer to the observed aspect ratio dependence. Numerical solution of the equations of motion in an ac field for the collective variables indicates that there is a critical ac amplitude where the curling direction will shift by multiples of π.

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I. INTRODUCTION AND MODEL DESCRIPTION

Nanoscale ferromagnetic disks have potential applications in magnetic information storage and microwave signal processing; therefore, there has been recent interest in the classical magnetic properties of confined magnetic systems. In this article the linear magnon excitations as well as the nonlinear dynamics of submicron ferromagnetic disks are investigated. These systems are of particular interest since submicron or micron size ferromagnetic disk can exhibit a vortex ground state as a result of competition between the exchange interaction and magnetostatic effects. The vortex state is characterized by two parameters, namely, the chirality of the vortex corresponding to clockwise or anticlockwise curling of the magnetization and the two possible directions of the relatively small (the order of the exchange length, $l_0 \approx 5$ nm for permalloy) out-of-plane vortex core. Since the vortex state is characterized by two parameters, this implies that a single disk can potentially store two bits of information with the additional requirement that these two parameters be externally controlled by an electric current or a magnetic field. For applications it is necessary to have an understanding of the low amplitude excitations on the vortex state as well as the higher amplitude excitations that are involved in the switching processes. In the following the dynamic properties of excitations on the vortex ground state are investigated using a combination of analytical and numerical techniques. The analytical work presented here is based on the Lagrangian for the ferromagnet[1] with collective variables where one variable is the generalized momentum, and Hamilton's equations will give the time dependence of these variables. Advantages of this technique are more accurate calculation of the spin wave mode frequencies than have been previously obtained from linearization of the Landau-Lifshitz equation, and simpler nonlinear equations of motion for the generalized coordinates. A disadvantage is the loss of information about the mode structure arising from the definition of the collective variables; however, nonlinear effects such as chirality switching are clearly evident.

Two types of dynamic excitations are typically produced in experimental setups. A magnetic pulse applied in the vortex plane will force the vortex center perpendicular to the field direction, and when the field is turned off the vortex core will undergo sub-GHz gyrotropic motion spiraling back to the disk center. This is the first type of vortex dynamics that was experimentally investigated[2] using time-resolved Kerr microscopy. Later higher frequency and lower amplitude modes were detected. A radially symmetric mode can also be produced by application of a magnetic pulse[3] perpendicular to the vortex plane, which has been observed through time-resolved Kerr microscopy[4,5] and Brillouin light scattering.[6] Because of the higher symmetry, the radially symmetric mode is now investigated using the collective variable approach and results are compared with earlier theoretical calculations. Past theoretical work[7] has indicated that the frequency of these excitations should be proportional to the square root of the aspect ratio, $\sqrt{L/R}$ where L and R are the thickness and radius of the disk, respectively. However, both experimental data and numerical simulations do not fit this simple square-root dependence very well. In this article a collective variable approach is used to obtain a different dependence of the excitation frequency that is a better fit to the experimental data.

Previously it has been demonstrated that vortex core magnetization direction can be switched by an alternating magnetic field[8,9] or by application of a spin-polarized current[10] in a nanocontact setup. Also there is the possibility[11] that the chirality can be switched by application of an ultrashort pulse. An advantage of the collective variable approach is the simplicity of Hamilton's equations, which are convenient for the description of forced oscillations of the magnetization under an alternating magnetic field along the vortex axis. Here it is shown by numerical solution of the dynamic equations that there is a critical field above which the chirality will switch. Moreover, the switching process is related to the development of a significant out-of-plane magnetization during the switching process.

In the following the generalized coordinates are defined based on deviations from the vortex ground state. The magnetic energy and Lagrangian as functions of the generalized coordinates are deduced, which leads to Hamilton's equations. The linearization of Hamilton's equations results in an expression for the spin wave frequency of these excitations as a function of the aspect ratio. Finally, switching as a result of an oscillating magnetic field as well as an ultrashort pulse are discussed.

To determine the form of the collective variables, we begin with the Lagrangian for the ferromagnet

$$\mathcal{L} = \frac{L}{\gamma}\int M_z \frac{\partial \phi}{\partial t} d^2r - W[\vec{M}], \tag{1}$$

where it was assumed that the magnetization in the thin ($R \gg L$) disk is independent of z resulting in a two-dimensional integral over the disk area, and $W[\vec{M}]$ is the energy of ferromagnet written as a functional of the magnetization, $\vec{M}$. It is convenient to express the vortex ground state in polar coordinates where the Cartesian components of the magnetization are $M_s(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ in terms of the azimuthal angle, φ and the polar angle, θ and M_s is the saturation magnetization. In a cylindrical (r,χ) coordinate system the vortex ground state is given by $\varphi=\chi+\pi/2$, and $\theta(r)=\pi/2$ over most of the disk, with $\theta\to 0,\pi$ as $r\to 0$ eliminating the exchange singularity at the vortex center. This is a small region referred to as the vortex core of radius the order of the exchange length where M_z is not negligible and in the following the core will only give small corrections to the calculated frequency. In the continuum approximation the energy is

$$W[\vec{M}] = L\int \left\{\frac{A}{2}\left[(\nabla\theta)^2 + \frac{\sin^2\theta}{r^2}\right] + 2\pi M_s^2\cos^2\theta\right\} d^2r + W_{\mathrm{vol}} + W_{\mathrm{edge}}, \tag{2}$$

where A is the nonhomogeneous exchange constant, the second term in the integrand describes the contribution from the magnetic charges at the two dot faces, where the z direction is defined by the disk axis, treated in the local "thin capacitor approximation" (see Ref. 12). The last two terms W_{vol} and W_{edge} are the contributions to the magnetostatic energy from the volume magnetostatic charge $(\nabla\cdot\vec{M})$ and edge magnetostatic charge $[M_r(R)]$, respectively, for a disk of radius R.

II. COLLECTIVE VARIABLES AND LINEAR OSCILLATIONS

To find a set of collective variables we begin with the Lagrangian of the ferromagnet where a variable, ψ representing the azimuthal deviation from the vortex ground state $\varphi=\chi+\pi/2+\psi(t)$ is introduced. Thus, the variable $\psi(t)$ is used as a collective coordinate describing azimuthal distortion of the vortex structure. It is worth noting here, the deviation of the magnetization is coordinate dependent, with the dependence determined by the out-of-plane component of magnetization $\vec{M}$ (see below). Next, placing the coordinate—independent quantity $\psi(t)$ outside the integral, we can write the kinetic part of the Lagrangian Eq. (1) as $(\mu/\gamma)(\partial\psi/\partial t)$, where the generalized coordinate μ is defined as

$$\mu = LM_s\int \cos\theta d^2r. \tag{3}$$

Thus, rather than using as a variable the polar angle θ, which depends on coordinates within the disk, we naturally arrive at the integral characteristic, μ. Then the Lagrangian for collective variables ψ and μ takes the form,

$$\mathcal{L} = \frac{1}{\gamma}\mu\frac{d\psi}{dt} - W(\mu,\psi), \tag{4}$$

where $W(\mu,\psi)$ is the effective energy depending on ψ and μ, and μ/γ is the generalized momentum conjugated with the coordinate ψ. According to the spirit of the collective variable method, the function $W(\mu,\psi)$ is equal to the energy functional of the ferromagnet $W[\vec{M}]$ [as in Eq. (1)], in which the minimization over the distribution of the magnetization $\vec{M}$ is performed under the condition that the values of ψ and μ are fixed. In terms of collective variables $W(\mu,\psi)$ plays the role of the Hamilton function, and the corresponding Hamilton equations have the form

$$\frac{1}{\gamma}\dot{\mu} = -\frac{\partial W}{\partial\psi} \tag{5}$$

$$\frac{1}{\gamma}\dot{\psi} = \frac{\partial W}{\partial\mu}. \tag{6}$$

To proceed it is necessary to find the dependence of the energy on the two generalized coordinates. According to the method of collective variables, $W(\mu,\psi)$ is a functional of the ferromagnetic energy $W(\vec{M})$, in which minimization over the magnetization distribution is done subject to the condition that ψ and μ are fixed.

First we determine the dependence of the energy on ψ for the thin disk in the vortex ground state. From Eq. (2) and the radial dependence of the excitations it is remarked that there is no angular dependence in the integral corresponding to the easy-plane ferromagnet. Therefore, the contributions are determined by the nonlocal terms resulting from edge and volume magnetostatic charge. The contribution from the edge charges have been estimated by asymptotic evaluation of the magnetostatic integral, and can be written as[12]

$$W(\psi) \approx 2\pi RL^2M_s^2\sin^2\theta(R)\ln\frac{\eta R}{L}\sin^2\psi, \tag{7}$$

where η is a parameter the order of 4 and the value $\sin^2\theta(R)$ on the lateral boundary of the disk, i.e., the circle of radius R. Then for the volume charges one can easily find

$$\nabla\cdot\vec{M} = [(1/r)d(r\sin\theta)/dr]\cos(\varphi-\chi),$$

and in the main part of the disk outside the vortex core, where $\theta=\theta(R)$, the density of the volume charges $\nabla\cdot\vec{M}$ is proportional to $\sin\theta(R)\sin\psi$. Then it can be shown that the

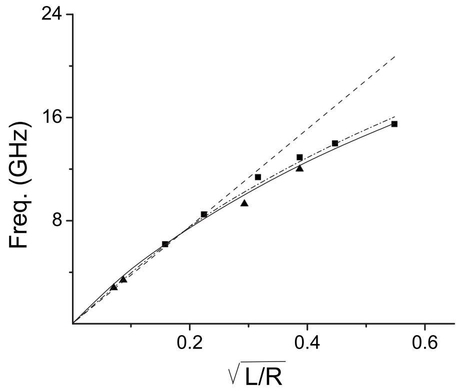

FIG. 1. Frequency versus the square root of the aspect ratio. The dashed line is the linear dependence. Squares represent simulation data. Triangles represent experimental data. The solid curve is the dependence given by Eq. (13) with $\eta=4$ and the dot-dash curve is the dependence given by Eq. (13) with $\eta=6$. The dashed curve illustrates the linear dependence.

contribution of the volume charges is the order of $RL^2M_s^2 \sin^2 \theta(R)\sin^2 \psi$. Therefore, although the inclusion of this contribution is significant, it is reduced to the renormalization of the coefficient η. As a result, the dependence of the energy on ψ includes both surface and volume magnetostatic charge and can be presented in the form (7), where the number $\eta \geq 4$ can be considered a phenomenological coefficient. As we will see below, the final values of the dynamical parameters are not too sensitive to the value of η, and all known data for the frequencies of radially symmetric modes, found experimentally, or through the numerical simulations, can be accurately described by use of the value $\eta=(4-6)$. Note that the logarithmic multiplyer is large up to the values of $L \leq (0.3-0.5)\cdot R$ where the theory developed here for thin disks can be valid. Thus, the dependence of the energy of the system on ψ is described by Eq. (7) with the proper value of η. As one can see from Fig. 1, the concrete choice of η in the range of a several units do not affects strongly on the predicted dependence of frequency on aspect ratio.

To determine the μ dependence of the energy, it is necessary to calculate the energy minimum of the system at a given value of μ. This done by the method of indefinite Lagrange multipliers to find the minimum of the functional, $\tilde{W}=W-H\mu$, where H is the Lagrange multiplier. It is remarked that the Lagrange multiplier enters the problem as an external magnetic field in the z direction. Therefore, the extremals of the function $\tilde{W}$ describe the magnetization distribution in the presence of such a field, which is the so-called cone state[13] vortex with the polar angle depending on H. The dependence on r and H is determined by solution of the equation

$$A\left(\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-\frac{1}{r^2}\sin\theta\cos\theta\right)+4\pi M_s^2(\sin\theta\cos\theta - h\sin\theta)=0, \tag{8}$$

where $h=H/4\pi M_s$. This has been investigated numerically[14] showing that there is a vortex solution for $|h|<1$, and the core size is an increasing function of h and becomes unrestricted as $h \to 1$. The value of $\theta(r)$ far from the core is determined by

$$\cos\theta = h\left(1+\frac{r_0^2}{r^2}\right) \tag{9}$$

and $r_0=\sqrt{A/4\pi M_s^2}$ is the exchange length.

These results for the cone state are used in Eq. (2) together with the relation between μ and h given by Eq. (3) to obtain $W(\mu)$. As a result the energy can be expressed as

$$W(\mu)=\mu H+\pi AL(1-h^2)\ln\left(\frac{R}{r_0}\sqrt{\pi\Lambda(h)}\right)-\pi R^2L(2\pi M_s^2h^2). \tag{10}$$

Here the function, $\Lambda(h)\approx 5.27$ as $h\to 0$ and linearly decreases to zero as $h \to 1$.[14]

To calculate an analytical expression for the excitation frequency, μ is eliminated from Hamilton's equations resulting in a second-order differential equation for ψ. When this is done it is necessary to evaluate the derivative, $d^2W/d\mu^2$. To the lowest order, this quantity can be evaluated at the equilibrium value of the magnetization, and using the first order approximation relating μ and h, $\mu=\pi M_s LR^2 h$ along with $dW/d\mu=H$ gives

$$\frac{d^2W}{d\mu^2}\cong\frac{4}{R^2L} \tag{11}$$

with higher-order corrections the order of r_0^2/R^2. Using these it is possible to express the equation for $\psi(t)$ in the standard form

$$\frac{d^2\psi}{dt^2}+\omega_0^2(1-h^2)\sin\psi\cos\psi=0 \tag{12}$$

where

$$\omega_0=\omega_M\sqrt{\frac{L}{\pi R}\ln\left(\frac{\eta R}{L}\right)}, \tag{13}$$

and $\omega_M=4\pi\gamma M_s$ is the characteristic frequency of the ferromagnetic material, $\omega_M\approx 30$ GHz for permalloy. The frequencies of the radiallly symmetric mode in the linear approximation are given by Eq. (13) where it is noticed that there is a relatively simple dependence on the disk aspect ratio.

Previous theoretical work[6] indicated a $\sqrt{L/R}$ frequency dependence, but experimental measurements and numerical simulations showed a deviation from this dependence. The calculated frequency here contains the additional logarithmic multiplier resulting in a better fit to experiment and simulations. Figure 1 illustrates the relation between Eq. (13), previous frequency calculations from micromagnetic simulations,[6] and previous experimental data.[2–5] In Fig. 1 the solid curve is the dependence of frequency on $\sqrt{L/R}$ from Eq. (13) with $\eta=4$, where the deviation from linearity is noticed at higher values of the aspect ratio. The squares represent numerical calculations[7] of the frequency with $R=200$ nm and varying L. The triangles represent experimental data obtained using permalloy disks of aspect ratios

0.005 ($L=15$ nm, $R=3000$ nm),[4] 0.0075 ($L=15$ nm, $R=2000$ nm),[5] 0.086 ($L=30$ nm, $R=3000$ nm),[3] and 0.15 ($L=30$ nm, $R=350$ nm).[6] The dashed line is a plot of $\omega=1.3\omega_M\sqrt{L/R}$. Notice that there is significant deviation from linearity for both experimental data and numerical simulations. Moreover, both sets of point are well fit by Eq. (13) for all aspect ratios considered. Finally, to illustrate the weak dependence of Eq. (13) on the possible values of η, the frequency from Eq. (13) with $\eta=6$ is indicated by the dot-dash curve.

III. GENERALIZATIONS

Notice that Eq. (12) also indicates that the frequency of the radially symmetric mode depends on an external field perpendicular to the vortex plane with the simple dependence given by

$$\omega_0(H)=\omega_0\sqrt{1-h^2}, \quad h=\frac{H}{4\pi M_s}.$$

This dependence is weak for a small field values, but becomes quite strong as $h\to 1$. It is remarked that the frequently observed second important mode describing vortex precession has the opposite behavior; namely, its frequency increases linearly with magnetic field, $\omega_{VP}(h)=\omega_{VP}(0)(1+h)$.[13,14] For this reason, the unperturbed frequencies should coincide at some value of the field h_0, $(1-h_0)=[\omega_{VP}(0)/\omega_0]^2(1+h_0)\approx[\omega_{VP}(0)/\omega_0]^2$, and one can expect the collectivization mode with the crossing avoidance near h_0. We are not aware of any experimental measurements or numerical simulations of the Landau-Lifshitz equation to verify this result.[15]

In past few years, some systems such as nanomagnets with intentionally inserted defects have received considerable attention, and a number of experimental and theoretical studies have been reported. First, the simplest structures like circular magnetic dots with a hole placed in the center (so-called magnetic rings) were proposed.[16] It has been shown, the vortices are more stable in such systems (see Ref. 17 for overview of first articles), and have some peculiar dynamical properties.[18,19] More complicated structures containing a noncentered hole[19–21] or even a few holes[22] have received considerable attention in the last years because of a number of potential applications (note the possibility of their use for types of magnetic logic).[22] On the other hand, only translational dynamics of the vortex core, which in fact means the investigation of the vortex core pinning by a defect, has been investigated theoretically.[23–25]

The approach developed here can be easily generalized for description of most symmetrical structures of this kind, such as magnetic rings, where the radial symmetry of the vortex is maintained. For this case, the main difference here are outer and inner radii R_o and $R_i<R_o$, respectively, and one must take into account the surface charges from both the inner and outer edges. In the thin ring approximation if both R_i and R_o, as well as their difference R_o-R_i are large enough, R_i, R_o, $R_o-R_i\gg L$, this contribution is the sum of two terms of the type (7). Then the ψ-dependent part of energy can be expressed as a sum

$$W^{(ring)}(\psi)=2\pi M_s^2L^2\left[R_o\sin^2\theta(R_o)\ln\left(\frac{\eta R_o}{L}\right)+R_i\sin^2\theta(R_i)\ln\left(\frac{\eta R_i}{L}\right)\right]. \tag{14}$$

In the thin ring approximation as above, the quantity $d^2W/d\mu^2$ depends on inner and outer radii of the ring. Thus, for the ring can this quantity can be obtained as

$$\left.\left(\frac{d^2W}{d\mu^2}\right)\right|_{\mu_0}=\frac{4}{(R_0^2-R_i^2)L}. \tag{15}$$

In fact, the ψ-dependent part of energy describes the energy from the "restoring force" during oscillation of ψ, and the quantity $1/(\gamma^2d^2W/d\mu^2)$ plays the role of the effective mass in the Hamilton equations for ψ. Thus, the oscillation of ψ for the case of a ring is described by the same Eq. (13), and the value of ω_0 is

$$\omega_0^{(ring)}=4\pi\gamma M_S\sqrt{\frac{L}{\pi(R_o-R_i)}\ln\left(\frac{\eta\bar{R}}{L}\right)+\frac{L}{2\pi(R_o+R_i)}\ln\left(\frac{R_o}{R_i}\right)}, \tag{16}$$

where $\bar{R}=\sqrt{R_oR_i}$. Here is noticed that the frequency increases as the size of the hole in the ring increases, or as R_0-R_i decreases. Since the theory developed is adequate for the condition $R_o-R_i\gg L$ only, Eq. (16) is valid for the case of $\omega_0^{(\mathrm{ring})}\ll 4\pi\gamma M_S$, but this limitation allows description of the oscillation even for the case of $\omega_0^{(\mathrm{ring})}\gg\omega_0$. Again, the simple estimates show, in the presence of the magnetic field the frequency is $\omega_0^{(\mathrm{ring})}(h)=\omega_0^{(\mathrm{ring})}\sqrt{1-h^2}$. It will be interesting to discover more general cases, with the hole displaced from the disk center, or even with a few holes in arbitrary positions, but such generalization is not so straightforward as above. Clearly, the radial geometry is now lost, and the simple choice of collective coordinates as above cannot be used. For one nonsymmetric hole, the two-vortex ansatz describing the vortex displaced to the center of the hole,[26] giving the fixed magnetization boundary conditions at both edges could be used. For a few holes the analysis will be more complex, but probably, for all these cases the effect of the hole will be the same as for simple ring structure: the frequency will be higher than for disk without hole. More detail analysis goes far from the scope of the article.

IV. NONLINEAR DYNAMICS

As well as a description of linear oscillations, Eq. (12) also describes nonlinear motion. For example, a characteristic nonlinear solution is $\cos\psi = \tanh(\omega_0 t)$ and $\sin\psi = 1/\cosh(\omega_0 t)$ describing a dynamic switch between energetically equivalent, but physically different states with $\psi = 0$ and $\psi = \pi$. Hamilton's Eqs. (5) and (6) are also convenient for the investigation of forced oscillations of the magnetization under an alternating magnetic field directed along the vortex axis. A few different cases will be considered: a rapid change in the value of magnetic field, an ultrashort pulse, and a monochromatic alternating field. For the first two cases it will be determined how large the amplitude of Ψ must be for switching to occur. For the third case of the ac field the field amplitude required for switching is determined.

To describe the effect of a rapid field change on the system (this method is frequently used in numerical simulations) it is sufficient to assume that before the field is switched on, $(t \le 0)$ a vortex state having some value of the total moment $(\mu = \mu_<)$ is realized, which differs from the total moment $(\mu = \mu_>)$ for $t > 0$ after the field is switched on. Then the magnetization evolution after the field is switched on is described by equations having the initial condition $\mu - \mu_0 = \Delta\mu = \mu_< - \mu_>$. For Hamilton's Eq. (6) for the variable ψ this can be described by the initial conditions, $\psi_{t=0} = 0$ and $(d\psi/dt)_{t=0} = \gamma\kappa\Delta\mu$. To estimate the effectiveness of this mechanism we consider oscillations of small amplitude, ψ_0 for which $\psi(t) = \psi_0 \sin\omega_0 t$. Using the relation between h and μ, $(\kappa\Delta\mu = \Delta H)$ the estimate of the oscillation amplitude, $\psi_0 = \gamma\Delta H/\omega_0$ is obtained. Therefore, since the frequency is the order of several GHz, a substantial field change is necessary. For example, for a typical $\Delta H \approx 100$ Oe and $\omega_0 \approx 10$ GHz the amplitude is $\psi_0 \approx 0.03$ and the oscillation does not exceed 5 degrees.

Next consider excitation of the system by a short pulse with amplitude H_0 and duration, $\Delta t \ll 1/\omega_0$ in which case the initial condition is $\psi_{t=0} = \gamma H_0 \Delta t$ and $(d\psi/dt)_{t=0} = 0$. Accordingly, within the linear approach oscillations are described by $\psi(t) = \psi_0 \cos\omega_0 t$, and the amplitude, $\psi_0 = \gamma H_0 \Delta t$ can be much larger. The value of the field in a pulsed laser beam can reach 1 T, and a pulse duration of 1 ps gives $\psi_0 \approx 0.2$. This result has been observed in numerical simulations and can result in $\psi_0 \approx 1$ eventually resulting in switching[27] of the variable, ψ.

The magnetization can also be switched by application[8,9] of an alternating magnetic field in which case it is best to numerically solve Hamilton's equations for both $\mu(t)$ and $\psi(t)$ since both of these variables can be rather large. Moreover, since larger amplitude nonlinear effects are considered it is necessary to include Gilbert damping with the damping parameter, α. First it is convenient to rescale the variable μ to define $y(t) = \mu(t)/\mu_{\max}$, where $\mu_{\max}$ is the maximum possible value when $\cos\theta = 1$. Then the equations of motion are

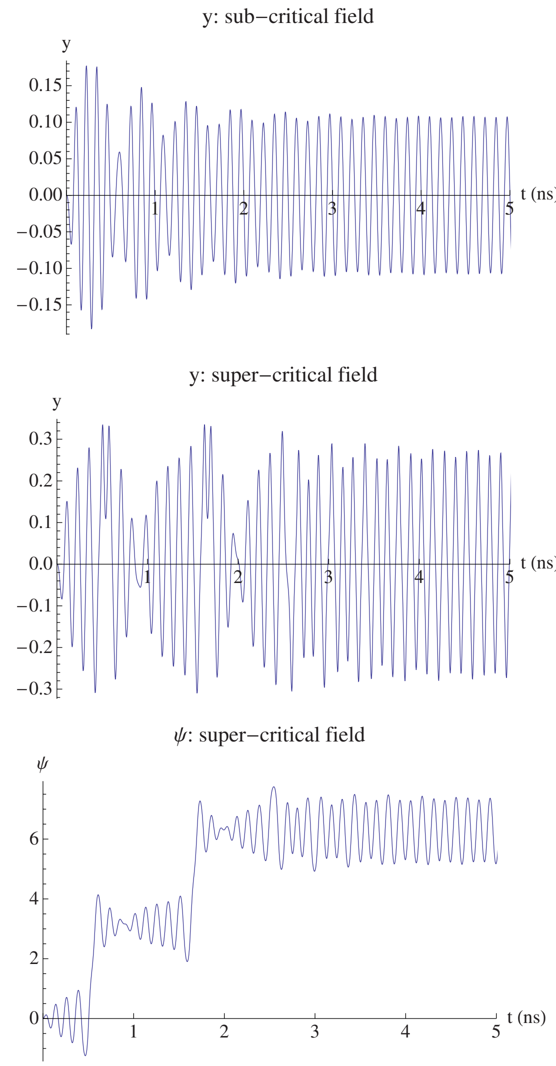

FIG. 2. (Color online) Top panel: $y(t)$ for the driving field below the critical field. Middle panel: $y(t)$ for the driving field slightly above the critical field. Lower panel: $\psi(t)$ for the driving field slightly above the critical field showing π transitions.

$$-\frac{dy}{dt} = \frac{L}{R}\frac{\omega_M}{\pi}(1-y^2)\ln\left(\frac{\eta L}{R}\right)\sin\psi\cos\psi + \alpha(1-y^2)\frac{d\psi}{dt} \tag{17}$$

$$\frac{d\psi}{dt} = \omega_M y - y\frac{L}{R}\frac{\omega_M}{\pi}\ln\left(\frac{\eta L}{R}\right)\sin^2\psi - \omega_M h_\sim \sin\omega t + \frac{\alpha}{1-y^2}\frac{dy}{dt}, \tag{18}$$

where $h_\sim = H_\sim/4\pi M_s$ is the dimensionless amplitude of the ac field of frequency, ω and α is a dimensionless damping constant, $\alpha \approx 0.01$ for permalloy. Having in mind the possibility of nonsmall values of $y(t) = \mu(t)/\mu_{\max}$ here we restored the multipliers $(1-y^2)$ in dynamical and relaxation terms, but, analysis shows, their role in minimal and do not essentially change the solution behavior. We would like to note here, for the numerical integration of the Eqs. (17) and (18) we choose the initial conditions $y=0$ and $\psi=0$ at the field switching on time, $t=0$. Physically, this corresponds to the natural condition with the system at rest before the force is applied. Moreover, the choice of the phase of the driving force, $h \propto \sin(\omega t)$, with the zero force at $t<0$, corresponds to the condition that the force is switched on smoothly. For this reason, we do not discuss how other initial conditions, as well as the phase of the force, have an effect on switching.

Numerical solution of Eqs. (17) and (18) with $h_{\sim}=0$ results in oscillatory $y(t)$ and $\psi(t)$ of frequency given by Eq. (13). At a driving frequencies in the GHz range the oscillations will remain periodic, but with a complicated beat pattern owing to the two distinct frequencies (ω and ω_0) and the nonlinearity. A typical time-dependent oscillation for small driving field is illustrated in the top panel of Fig. 2, where it is noticed that oscillations are about the values, $y=0$ and $\psi=0$. As the magnitude of the driving field increases, there is a critical value where the oscillations appear to become aperiodic and there are significant changes in the nature of the oscillations. At or above the critical field the variable, ψ undergoes transitions in steps of $n\pi$, where n is an integer. For the particular case when n is an odd integer this corresponds to a switch in vortex chirality, however, we have found that the nature of the switching exhibits sensitive dependence on the value of field amplitude. In general, slightly above the critical field during a short time interval there can be series of $n\pi$ transitions and after a time of a few ns damping will tend to lock ψ onto an integral value of π. At particular values of h there can be a "staircase" of transitions, and a very small change in h will result in a single transition. This is illustrated in the second panel of Fig. 2 where there is a 2π transition, which is stable after a few ns.

In the planar ($\theta=\pi/2$) vortex state disk chirality switch is inhibited by the large increase in magnetostatic energy as ψ rotates from $\chi+\pi/2$ to $\chi+3\pi/2$, but the edge charge magnetostatic energy is minimized by development of an out-of-plane magnetization at the disk edge. For this reason it is expected that there is a significant shift in the direction of the edge magnetization during the transition. Due to the nature of the collective variable, y defined by an integral over the disk, it is not possible to obtain the radial dependence of M_z. However, it is noticed that there is a significant out-of-plane magnetization during this transition where $y(t)$ oscillates about a nonzero value. This effect is seen in the lower panel of Fig. 2, where it is also noticed that the oscillation is about a nonzero value ($\bar{y} \cong 0.25$) for a few cycles during the transition. Note all the sharp changes in solution appears at some initial time interval, of order of (15–20) periods of oscillations, that is, of order of $2\pi/\alpha\omega$. This demonstrates the importance of the "free" oscillations of the system, on the other words, the importance of the initial conditions used.

In summary, the generalized coordinate approach applied to the vortex state disk results in calculated frequencies in the linear approximation agreeing very well with those obtained experimentally as well as through simulation of the Landau-Lifshitz equation for these systems. Numerical solution of the nonlinear dynamical equations shows that there will be a critical driving field at which the curling direction will switch by multiples of π. When the driving frequency is at the radial mode frequency there is a kink in the critical field, which is not yet understood, but is the subject of future research.

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